Missing Data Workshop
Joint Doctoral Program in Clinical Psyc

Overview of Multiple Imputation

Jonathan Lee Helm
Friday May 17th, 2019

Grand Overview

- Single Imputation
- Multiple Imputation
- The Imputation Process
- When does Multiple Imputation work?
- A note about Assumptions

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Single Imputation

Single Imputation

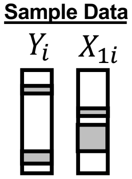
- Replace missing values with a 'guess'
  - Different approaches for choosing the guess

Single Imputation

- Replace missing values with a 'guess'
  - *Creates a complete data set*
    - *Different approaches for choosing the guess*
- Analyze complete data

Conceptual Diagram of Single Imputation

Sample Data



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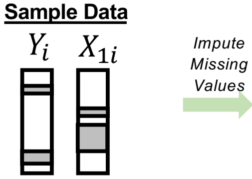

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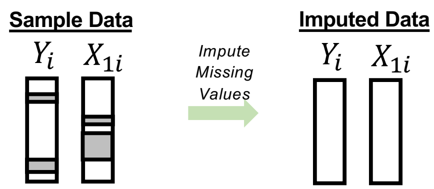
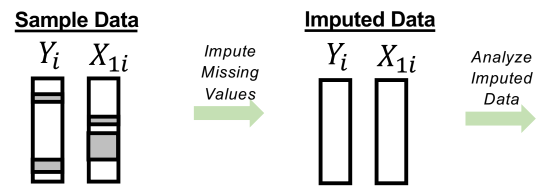
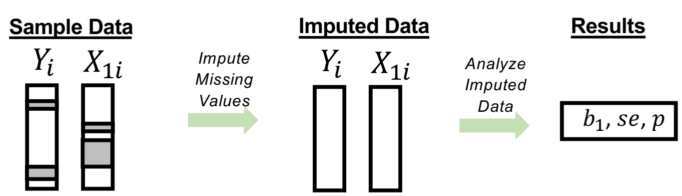


Conceptual Diagram of Single Imputation



Conceptual Diagram of Single Imputation



Single Imputation

- How can we create a guess?

Single Imputation

- How can we create a guess?
- Conceptually, the simplest way is through regression

Observed Data

i	JS^{obs}	IQ^{obs}
1	--	78
2	--	84
3	--	84
4	--	85
5	--	87
6	--	91
7	--	92
8	--	94
9	--	94
10	--	96
11	7	99
⋮	⋮	⋮
19	16	118
20	12	134

Observed Data

i	JS^{obs}	IQ^{obs}
1	--	78
2	--	84
3	--	84
4	--	85
5	--	87
6	--	91
7	--	92
8	--	94
9	--	94
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11	7	99
⋮	⋮	⋮
19	16	118
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Regression:

$JS^{obs} = b_0 + b_1 IQ^{obs}$

Observed Data

i	JS^{obs}	IQ^{obs}
1	--	78
2	--	84
3	--	84
4	--	85
5	--	87
6	--	91
7	--	92
8	--	94
9	--	94
10	--	96
11	7	99
⋮	⋮	⋮
19	16	118
20	12	134

Regression:

$JS^{obs} = b_0 + b_1 IQ^{obs}$

	Est.	s.e.	p
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

Observed Data

i	JS^{obs}	IQ^{obs}
1	--	78
2	--	84
3	--	84
4	--	85
5	--	87
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7	--	92
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Regression:

$JS^{obs} = b_0 + b_1 IQ^{obs}$

	Est.	s.e.	p
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

Imputed Data

i	JS^{imp}	IQ^{imp}
1	7.56	78
2	8.31	84
3	8.31	84
4	8.43	85
5	8.68	87
6	9.17	91
7	9.29	92
8	9.54	94
9	9.54	94
10	9.79	96
11	7	99
⋮	⋮	⋮
19	16	118
20	12	134

Observed Data

i	JS^{obs}	IQ^{obs}
1	--	78
2	--	84
3	--	84
4	--	85
5	--	87
6	--	91
7	--	92
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Imputed Data

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⋮	⋮	⋮
19	16	118
20	12	134

Mean for JS^{obs} = 11.7
SD for JS^{obs} = 2.71

Mean for JS^{imp} = 10.28
SD for JS^{imp} = 2.42

Complete Data

i	JS^{com}	IQ^{com}
1	9	78
2	13	84
3	10	84
4	8	85
5	7	87
6	7	91
7	9	92
8	9	94
9	11	94
10	7	96
11	7	99
⋮	⋮	⋮
19	16	118
20	12	134

Imputed Data

i	JS^{imp}	IQ^{imp}
1	7.56	78
2	8.31	84
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4	8.43	85
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Mean for JS^{obs} = 11.7
SD for JS^{obs} = 2.71

Mean for JS^{imp} = 10.28
SD for JS^{imp} = 2.42

Mean for JS^{com} = 10.35
SD for JS^{com} = 2.68

Single Imputation

- We impute values based on the observed data
- This will work well when data are MCAR or MAR, but not MNAR

Single Imputation: MCAR

- Is single imputation reasonable?
- If data are MCAR, then the imputed values will just be random guesses
  - *This should not impact parameter estimates*
  - *We will use a larger sample size, so decreased standard errors*

Single Imputation: MAR

- Is single imputation reasonable?
- If the data are MAR, then the other variables in the model that relate to missingness will create good predicted values
  - *This should create less biased parameter estimates*
  - *Increase sample size*

Single Imputation: MNAR

- Is single imputation reasonable?
- If the data are MNAR, then the other variables in the analysis won't account for missingness
  - *This won't fully account for the bias*

Single Imputation: Limitation

- The limitation is that we are not accounting for the uncertainty of the regression

Single Imputation

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Regression:

$JS^{obs} = b_0 + b_1 IQ^{obs}$

$\sigma_\varepsilon^2 = 2.95$
$\sigma_\varepsilon = 1.72$

	Est.	s.e.	*p*
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

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Single Imputation

- We can take certainty into accounted by creating multiple data sets

Regression:

$JS^{obs} = b_0 + b_1 IQ^{obs}$

$\sigma_\varepsilon^2 = 2.95$
$\sigma_\varepsilon = 1.72$

	Est.	s.e.	*p*
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

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Single Imputation

- We can take certainty into accounted by creating multiple data sets (***Multiple imputation***)

Regression:

$JS^{obs} = b_0 + b_1 IQ^{obs}$

$\sigma_\varepsilon^2 = 2.95$
$\sigma_\varepsilon = 1.72$

	Est.	s.e.	*p*
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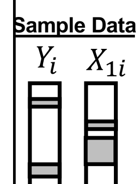
Multiple Imputation

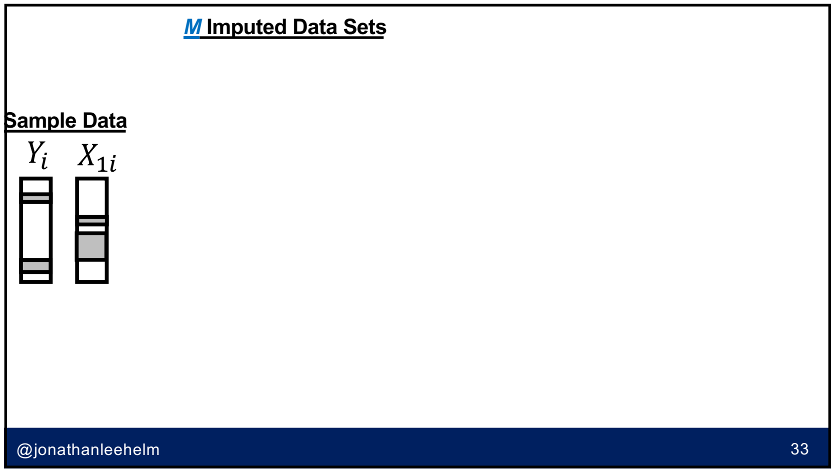
Multiple Imputation

- Multiple imputation extends single imputation by creating/analyzing more than one imputed data set

Multiple Imputation

- Multiple imputation extends single imputation by creating/analyzing more than one imputed data set
- We create *M* imputed data sets
- Each data set includes some uncertainty for the imputed value




M Imputed Data Sets
Sample Data
Y_i X_{1i}
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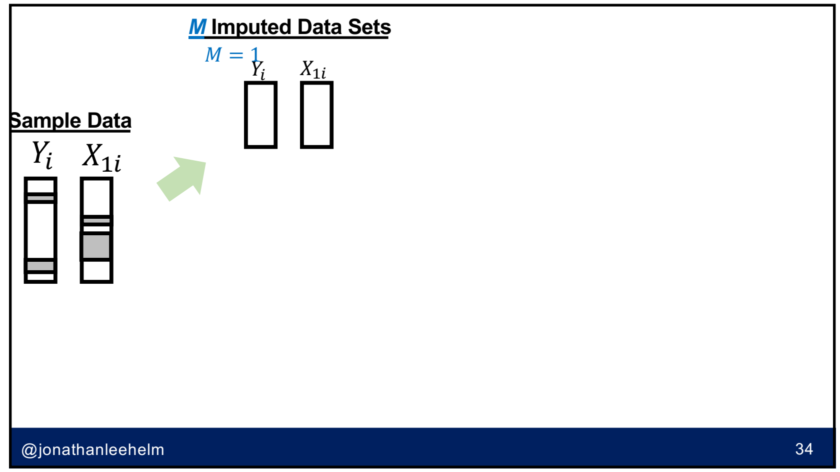
M Imputed Data Sets
$M = 1$
Y_i X_{1i}
Sample Data
Y_i X_{1i}
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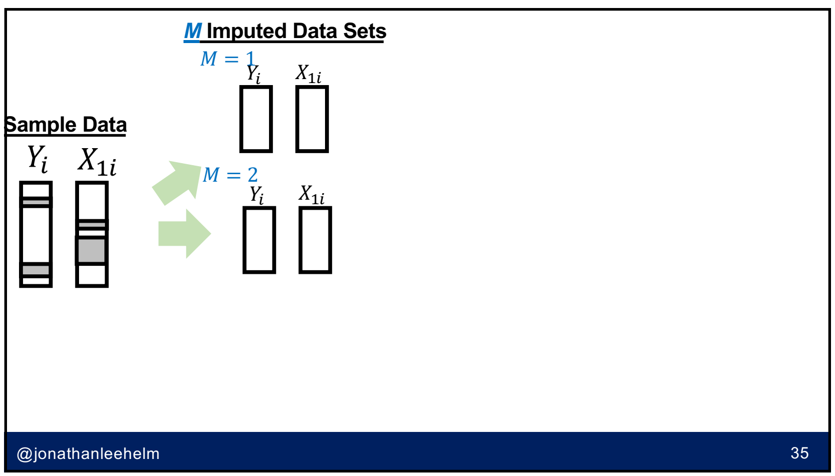
M Imputed Data Sets
$M = 1$
Y_i X_{1i}
Sample Data
Y_i X_{1i}
$M = 2$
Y_i X_{1i}
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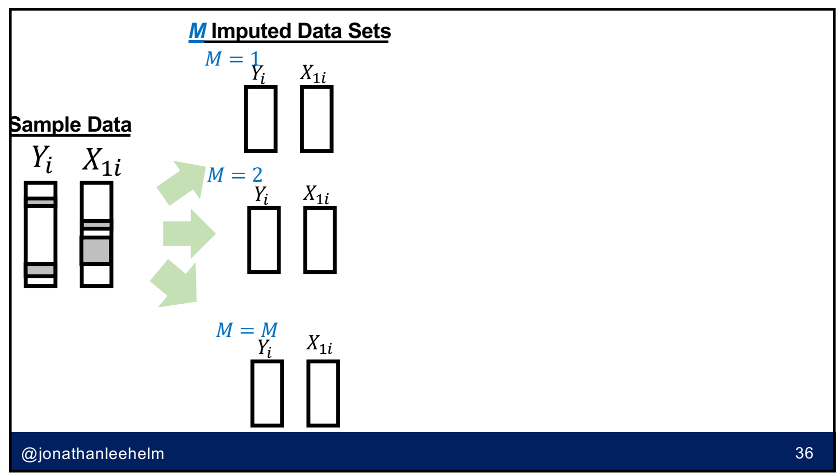
M Imputed Data Sets
$M = 1$
Y_i X_{1i}
Sample Data
Y_i X_{1i}
$M = 2$
Y_i X_{1i}
$M = M$
Y_i X_{1i}
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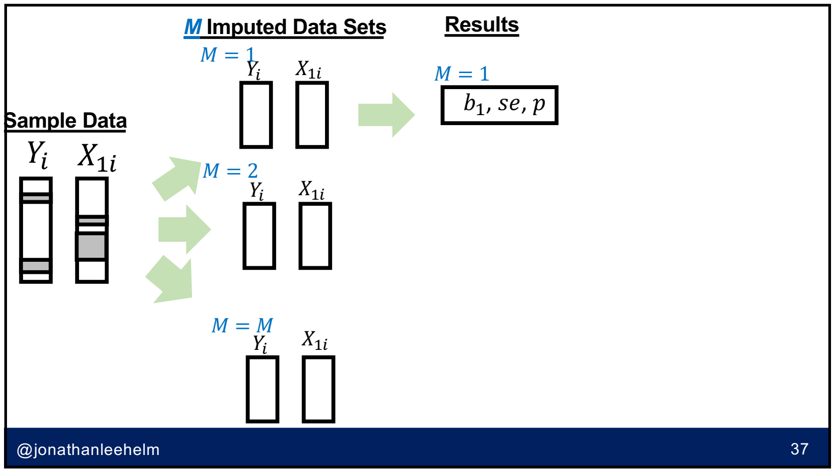
M Imputed Data Sets
Results
Sample Data
Y_i X_{1i}
$M = 1$
$M = 2$
$M = M$
b_1, se, p
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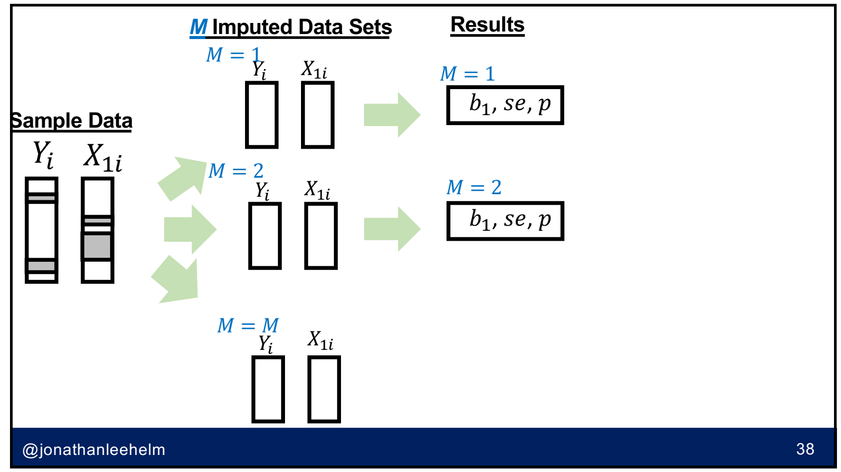
M Imputed Data Sets
Results
Sample Data
Y_i X_{1i}
$M = 1$
$M = 2$
$M = M$
b_1, se, p
b_1, se, p
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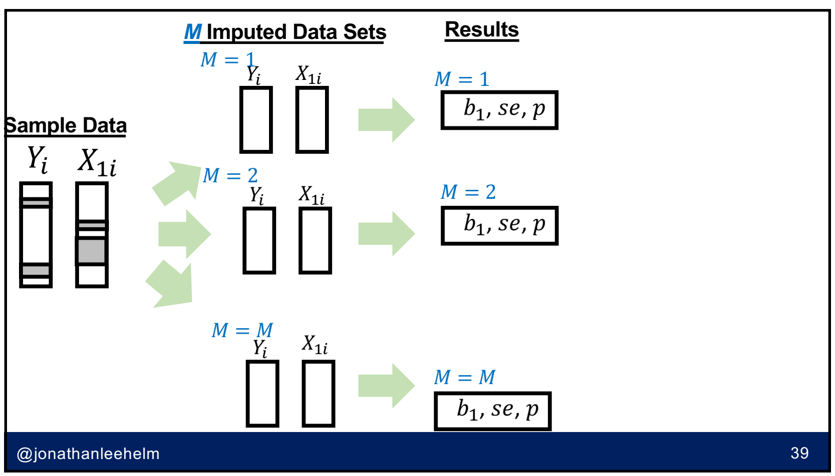
M Imputed Data Sets
Results
Sample Data
Y_i X_{1i}
$M = 1$
$M = 2$
$M = M$
b_1, se, p
b_1, se, p
b_1, se, p
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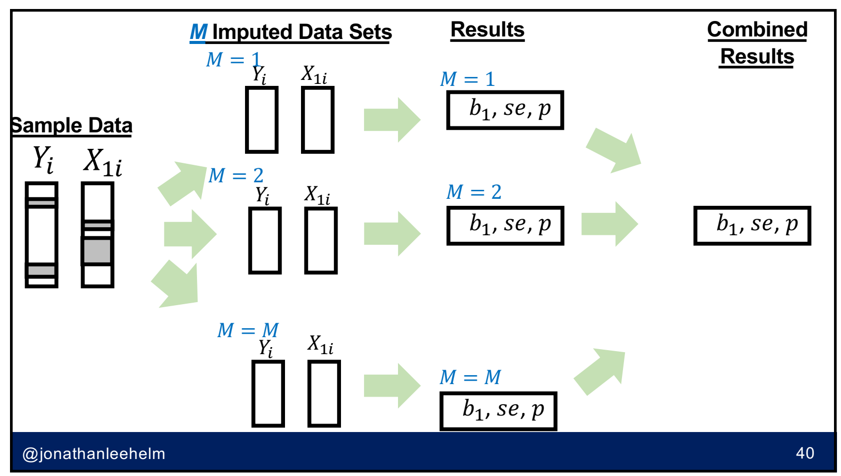
M Imputed Data Sets
Results
Combined Results
Sample Data
Y_i X_{1i}
$M = 1$
$M = 2$
$M = M$
b_1, se, p
b_1, se, p
b_1, se, p
b_1, se, p
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Observed Data		
i	JS^{obs}	IQ^{obs}
1	--	78
2	--	84
3	--	84
4	--	85
5	--	87
6	--	91
7	--	92
8	--	94
9	--	94
10	--	96
11	7	99
⋮	⋮	⋮
19	16	118
20	12	134

Observed Data			Imp Data M = 1		
i	JS^{obs}	IQ^{obs}	*i*	JS^{imp}	IQ^{imp}
1	--	78	1	15	78
2	--	84	2	7	84
3	--	84	3	10	84
4	--	85	4	10	85
5	--	87	5	15	87
6	--	91	6	11	91
7	--	92	7	10	92
8	--	94	8	15	94
9	--	94	9	10	94
10	--	96	10	10	96
11	7	99	11	7	99
⋮	⋮	⋮	⋮	⋮	⋮
19	16	118	19	16	118
20	12	134	20	12	134

Observed Data			Imp Data M = 1			Imp Data M = 2		
i	JS^{obs}	IQ^{obs}	*i*	JS^{imp}	IQ^{imp}	*i*	JS^{imp}	IQ^{imp}
1	--	78	1	15	78	1	11	78
2	--	84	2	7	84	2	7	84
3	--	84	3	10	84	3	10	84
4	--	85	4	10	85	4	10	85
5	--	87	5	15	87	5	10	87
6	--	91	6	11	91	6	10	91
7	--	92	7	10	92	7	7	92
8	--	94	8	15	94	8	15	94
9	--	94	9	10	94	9	11	94
10	--	96	10	10	96	10	15	96
11	7	99	11	7	99	11	7	99
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
19	16	118	19	16	118	19	16	118
20	12	134	20	12	134	20	12	134

Observed Data			Imp Data M = 1			Imp Data M = 2			Imp Data M = 3		
i	JS^{obs}	IQ^{obs}	*i*	JS^{imp}	IQ^{imp}	*i*	JS^{imp}	IQ^{imp}	*i*	JS^{imp}	IQ^{imp}
1	--	78	1	15	78	1	11	78	1	7	78
2	--	84	2	7	84	2	7	84	2	7	84
3	--	84	3	10	84	3	10	84	3	15	84
4	--	85	4	10	85	4	10	85	4	10	85
5	--	87	5	15	87	5	10	87	5	10	87
6	--	91	6	11	91	6	10	91	6	10	91
7	--	92	7	10	92	7	7	92	7	10	92
8	--	94	8	15	94	8	15	94	8	7	94
9	--	94	9	10	94	9	11	94	9	10	94
10	--	96	10	10	96	10	15	96	10	7	96
11	7	99	11	7	99	11	7	99	11	7	99
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
19	16	118	19	16	118	19	16	118	19	16	118
20	12	134	20	12	134	20	12	134	20	12	134

Combining Results

- Perform the analysis on each data set separately
- Collect the statistics of interest
  - *q*
    - *e.g., Mean, SD, regression coefficient*
- Calculated the average across the statistics

Observed Data

i	JS^obs	IQ^obs
1	--	78

	Est.	s.e.	*p*
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

i	JS^obs	IQ^obs
6	--	91
7	--	92
8	--	94
9	--	94
10	--	96
11	7	99
⋮	⋮	⋮
19	16	118
20	12	134

Imp Data M = 1

i	JS^imp	IQ^imp
1	15	78
2	7	84
3	10	84
4	10	85
5	15	87
6	11	91
7	10	92
8	15	94
9	10	94
10	10	96
11	7	99
⋮	⋮	⋮
19	16	118
20	12	134

Imp Data M = 2

i	JS^imp	IQ^imp
1	11	78
2	7	84
3	10	84
4	10	85
5	10	87
6	10	91
7	7	92
8	15	94
9	11	94
10	15	96
11	7	99
⋮	⋮	⋮
19	16	118
20	12	134

Imp Data M = 3

i	JS^imp	IQ^imp
1	7	78
2	7	84
3	15	84
4	10	85
5	10	87
6	10	91
7	10	92
8	7	94
9	10	94
10	7	96
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⋮	⋮	⋮
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Observed Data

i	JS^obs	IQ^obs
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	Est.	s.e.	*p*
b_0	-2.06	9.92	.84
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6	--	91
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8	--	94
9	--	94
10	--	96
11	7	99
⋮	⋮	⋮
19	16	118
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Imp Data M = 1

i	JS^imp	IQ^imp
1	15	78
2	7	84
3	10	84

	Est.	s.e.	*p*
b_0	5.09	4.22	.24
b_1	.064	.04	.14

i	JS^imp	IQ^imp
8	15	94
9	10	94
10	10	96
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⋮	⋮	⋮
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Imp Data M = 2

i	JS^imp	IQ^imp
1	11	78
2	7	84
3	10	84
4	10	85

	Est.	s.e.	*p*
b_0	4.64	3.03	.14
b_1	.064	.03	.05

i	JS^imp	IQ^imp
9	11	94
10	15	96
11	7	99
⋮	⋮	⋮
19	16	118
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Imp Data M = 3

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	Est.	s.e.	*p*
b_0	4.09	2.99	.19
b_1	.069	.03	.03

Observed Data

	Est.	s.e.	*p*
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

Imp Data M = 1

	Est.	s.e.	*p*
b_0	5.09	4.22	.24
b_1	.064	.04	.14

Imp Data M = 2

	Est.	s.e.	*p*
b_0	4.64	3.03	.14
b_1	.064	.03	.05

Imp Data M = 3

	Est.	s.e.	*p*
b_0	4.09	2.99	.19
b_1	.069	.03	.03

$$\frac{5.09 + 4.64 + 4.09}{3} = 4.60$$

$$\frac{.064 + .064 + .069}{3} = .066$$

Pooled Estimates across Imputations

	Est.	s.e.	*p*
b_0	4.60	3.51	.21
b_1	.066	.034	.07

Combining Results

- What about standard errors?

Combining Results

- What about standard errors?
  - Within imputation variability (W):
    - Average the squares of the s.e. (variances within each imputation)
  - Between imputation variability (B):
    - Include the variance across imputation estimates

Observed Data	Est.	s.e.	p
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

Imp Data M = 1	Est.	s.e.	p
b_0	5.09	4.22	.24
b_1	.064	.04	.14

Imp Data M = 2	Est.	s.e.	p
b_0	4.64	3.03	.14
b_1	.064	.03	.05

Imp Data M = 3	Est.	s.e.	p
b_0	4.09	2.99	.19
b_1	.069	.03	.03

$$W = \frac{4.22^2 + 3.03^2 + 2.99^2}{3}$$

Pooled Estimates across Imputations

	Est.	s.e.	p
b_0	4.60	3.51	.21
b_1	.066	.034	.07

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Observed Data	Est.	s.e.	p
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

Imp Data M = 1	Est.	s.e.	p
b_0	5.09	4.22	.24
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Imp Data M = 3	Est.	s.e.	p
b_0	4.09	2.99	.19
b_1	.069	.03	.03

$$W = \frac{4.22^2 + 3.03^2 + 2.99^2}{3}$$

$$B = var(5.09, 4.64, 4.09)$$

Pooled Estimates across Imputations

	Est.	s.e.	p
b_0	4.60	3.51	.21
b_1	.066	.034	.07

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Observed Data	Est.	s.e.	p
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

Imp Data M = 1	Est.	s.e.	p
b_0	5.09	4.22	.24
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Imp Data M = 2	Est.	s.e.	p
b_0	4.64	3.03	.14
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Imp Data M = 3	Est.	s.e.	p
b_0	4.09	2.99	.19
b_1	.069	.03	.03

$$W = \frac{4.22^2 + 3.03^2 + 2.99^2}{3}$$

$$B = var(5.09, 4.64, 4.09)$$

$$V_{b_0} = W + \left(1 + \frac{1}{M}\right)B$$

Pooled Estimates across Imputations

	Est.	s.e.	p
b_0	4.60	3.51	.21
b_1	.066	.034	.07

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Observed Data	Est.	s.e.	p
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

Imp Data M = 1	Est.	s.e.	p
b_0	5.09	4.22	.24
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Imp Data M = 2	Est.	s.e.	p
b_0	4.64	3.03	.14
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Imp Data M = 3	Est.	s.e.	p
b_0	4.09	2.99	.19
b_1	.069	.03	.03

$$W = \frac{4.22^2 + 3.03^2 + 2.99^2}{3}$$

$$B = var(5.09, 4.64, 4.09)$$

$$V_{b_0} = W + \left(1 + \frac{1}{M}\right)B$$

$$se_{b_0} = \sqrt{V_{b_0}} = 3.51$$

Pooled Estimates across Imputations

	Est.	s.e.	p
b_0	4.60	3.51	.21
b_1	.066	.034	.07

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Observed Data

	Est.	s.e.	p
b_0	-2.06	9.92	.84
b_1	.123	.09	.20

Imp Data M = 1

	Est.	s.e.	p
b_0	5.09	4.22	.24
b_1	.064	.04	.14

Imp Data M = 2

	Est.	s.e.	p
b_0	4.64	3.03	.14
b_1	.064	.03	.05

Imp Data M = 3

	Est.	s.e.	p
b_0	4.09	2.99	.19
b_1	.069	.03	.03

$$W = \frac{.04^2 + .03^2 + .03^2}{3}$$

$$B = var(.064, .064, .069)$$

$$V_{b_0} = W + \left(1 + \frac{1}{M}\right) B$$

$$se_{b_0} = \sqrt{V_{b_0}} = 0.34$$

Pooled Estimates across Imputations

	Est.	s.e.	p
b_0	4.60	3.51	.21
b_1	.066	.034	.07

Multiple Imputation: Summary

- Take aways:
  - Multiple imputation extends single imputation by creating/analyzing more than one imputed data set
  - Each data set includes some uncertainty for the imputed value

Grand Overview

- Single Imputation
- Multiple Imputation
- The Imputation Process
- When does Multiple Imputation work?
- A note about Assumptions

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The Imputation Process

Technical Aspects of Imputation

- Multiple imputation software rarely uses multiple regression to impute values for missingness
- The actual process is a bit more technical, but it can be conceptually related to regression
  - *Hence the way I teach it*

Technical Aspects of Imputation

- Multiple imputation software rarely uses multiple regression to impute values for missingness
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  - *Hence the way I teach it*
- http://www.stat.columbia.edu/~gelman/arm/missing.pdf
- https://www.jstatsoft.org/article/view/v045i03/v45i03.pdf

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Multiple Imputation
When does it work?

When Does MI Perform Well?

- Multiple imputation will perform well if one of the variables in the imputation accounts for the missingness
  - *The data are missing at random (MAR)*

When Does MI Perform Well?

- Multiple imputation will perform well if one of the variables in the imputation accounts for the missingness
  - *The data are missing at random (MAR)*
- Multiple imputation will also perform well if the missingness is not related to any variable in the data set
  - *The data are missing completely at random (MCAR)*

When Does MI Perform Well?

- Multiple imputation will not perform well if the missingness cannot be accounted for by the data
  - *The data are missing not at random (MNAR)*

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Assumptions in
Scientific Inference

Statistics in Science

- Statistics are foundation underlying scientific evidence
- Statistics are the language scientists use to make arguments

Statistics and Assumptions

- Virtually all inferential statistics rely on assumptions

Statistics and Assumptions

- Virtually all inferential statistics rely on assumptions

1. Random sample from the population
2. Certain variables follow a normal distribution
3. No measurement error
4. Independent observations
5. Equal variance across groups (or across the regression line)
6. The model is correct in the population

Statistics and Assumptions

- Regression: $Y_i = b_0 + b_1 X_i + \varepsilon_i$
- $b_1 = 5,\ p = .001$

- Common interpretation:
  - X_i significantly affects Y_i in the population

Statistics and Assumptions

- Regression: $Y_i = b_0 + b_1 X_i + \varepsilon_i$; $b_1 = 5,\ p = .001$

- Correct interpretation:
  - If I have a random sample from the population
  - and X_i is measured without error
  - and $Y_i = b_0 + b_1 X_i$ is the true model (i.e., nothing else affects Y_i)
  - and ε_i actually follows a normal distribution
  - and the variance of ε_i is constant around Y_i
  - and all of the observations are independent
- Then if the null hypothesis is true (if b_1 actually equals 0 in the pop.), the probability of getting 5 (or a value more extreme) is equals .001

Statistics and Assumptions

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- Then if the null hypothesis is true (if b_1 actually equals 0 in the pop.), the probability of getting 5 (or a value more extreme) is equals .001

If we have missing data, then we need to add:
and the data are missing completely at random

Statistics and Assumptions

- Regression: $Y_i = b_0 + b_1 X_i + \varepsilon_i$; $b_1 = 5,\ p = .001$

- Correct interpretation:
  - If I have a random sample from the
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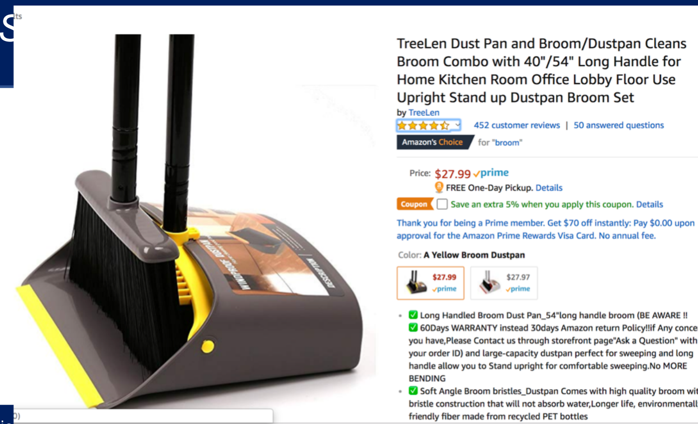
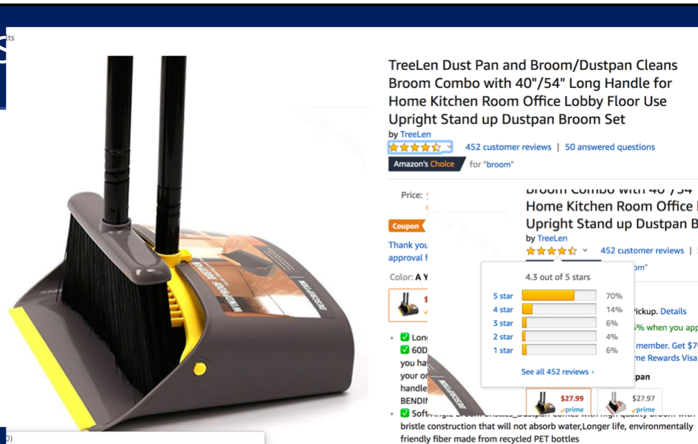
  - and ε_i actually follows a normal distribution
  - and the variance of ε_i is constant around Y_i
  - and all of the observations are independent
- Then if the null hypothesis is true (if b_1 actually equals 0 in the pop.), the probability of getting 5 (or a value more extreme) is equals .001

If we have missing data, and we performed multiple imputation:
and the data are missing at random

Statistics and Assumptions

- Don't panic





Statistics and Assumptions

- Interpretation:
  - If I buy the broom, I will be happy with it
- Correct interpretation:
  - If I have a random sample from the population
  - and ratings are measured without error
  - and ratings of the broom a direct reflection of broom satisfaction
    - *is the true model (i.e., nothing else affects Y_i)*
  - and all of the observations are independent
- Then I will be happy with the broom

Statistics and Assumptions

- Interpretation:
  - If I buy the broom, I will be happy with it
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    - *is the true model (i.e., nothing else affects Y_i)*
  - and all of the observations are independent
- Then I will be happy with the broom

These assumptions are likely not true

And I would still buy the broom

Statistics and Assumptions

- Don't panic
- The most important part is to recognize that assumptions that you're making when you're drawing conclusions
- Missing data mechanisms are a part of those assumptions
- So include it, and draw conclusions accordingly

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